Warm-Up

<u>Current: Alg 13.0</u>

Solve for *y* using two different methods.

$$\frac{5}{4}+\frac{3y}{2}=\frac{7y}{6}$$

<u>Review: 7NS 1.1</u>

Replace ◯ with $<, >$ **or** $=$ to make the sentence true.

$$\frac{5}{8} \bigcirc \frac{3}{4}$$

<u>Selected Response: CCSS 5.NF.1</u>

Which of these methods would be appropriate for finding $\frac{2}{5}+\frac{6}{15}$?

(A)

$$\frac{2}{5}+\frac{6}{15}$$
$$=\frac{2+6}{5+15}$$

(B)

$$\frac{2}{5}+\frac{6}{15}$$
$$=\frac{2}{5}(3)+\frac{6}{15}$$
$$=\frac{2}{15}+\frac{6}{15}$$
$$=\frac{2+6}{15}$$

(C)

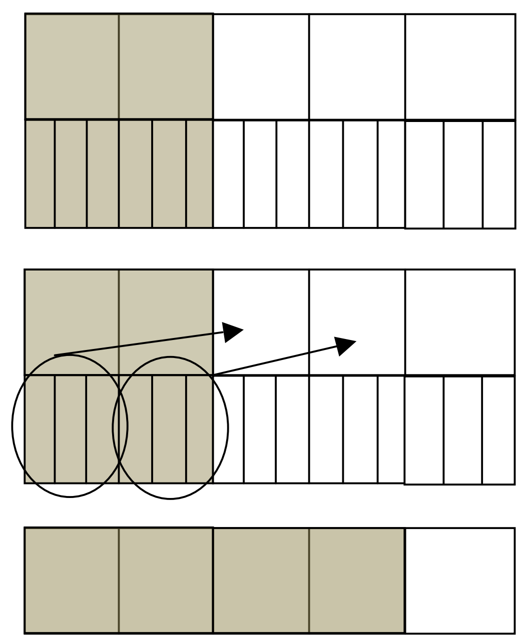
$$\frac{2}{5}+\frac{6}{15}$$
$$=\frac{1}{5}+\frac{1}{5}+\frac{6}{15}$$
$$=\frac{3}{15}+\frac{3}{15}+\frac{6}{15}$$
$$=\frac{3+3+6}{15}$$

(D)

$$\frac{2}{5}+\frac{6}{15}$$
$$=\frac{2}{5}\left(\frac{3}{3}\right)+\frac{6}{15}$$
$$=\frac{6}{15}+\frac{6}{15}$$

***Why would a student choose an incorrect answer choice?**

(E)



<u>**Lowest Common Multiple through the Grades**</u>

CA Standards: Algebra 12.0, 13.0, Grade 7 NS 1.2, 2.2, Grade 6 NS 2.1, 2.4
Common Core Standards: 6.NS.4, 7.NS.1, 1d, A.APR.1, A.SSE.2, A.SSE.3

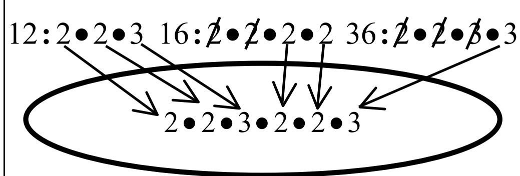
The Lowest Common Multiple (LCM) is a utility that can be used throughout multiple grade levels. In this lesson, we will look at ways we use the LCM in grades 5-7, in Algebra and Algebra II. More importantly, teaching students to use the LCM as a tool to solve math problems will help them going forward in their math careers.

Grades 5-7

In grades 5-7 what can we use Lowest Common Multiple (LCM) for? We most commonly see problems involving finding the LCM of two or three different numbers.

Example 1: Find the LCM of a set of numbers $12, 16, 36$

Bubble Method:



$\therefore$ **The LCM is 144**

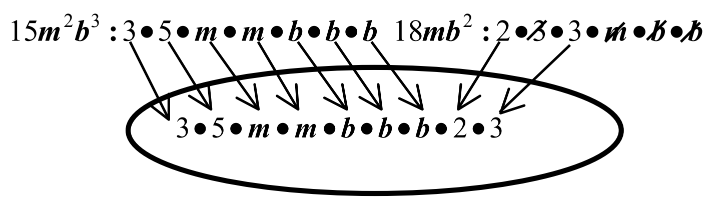
You Try!
Find the LCM of $12, 16,$ **and** 10

Algebra

In Algebra what do we use LCM for? Similar to the lower grades, we can use the Bubble Method to help find the LCM of a set of polynomials.

Example 1: Find the LCM of the set of polynomials $15m^2b^3$ and $18mb^2$

Bubble Method:



$\therefore$ **The LCM is** $90m^2b^3$

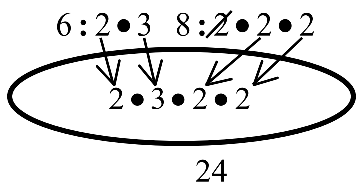
You Try!
Find the LCM of $28m^2n$ **and** $12m^2n^3p$

Grade 5-7

It can be used when working with unlike fractions. Text books will refer to the LCM as the Lowest Common denominator (LCD).

Example 2: Adding Fractions

Use the Bubble Method:



$$-\frac{1}{6}+\frac{5}{8}$$
$$=\left(-\frac{1}{6}\bullet\frac{4}{4}\right)+\left(\frac{5}{8}\bullet\frac{3}{3}\right)$$
$$=-\frac{4}{24}+\frac{15}{24}$$
$$=\frac{-4+15}{24}$$
$$=\frac{11}{24}$$

You Try!

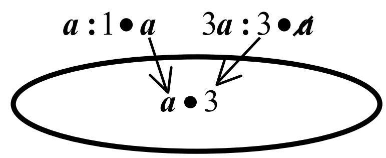
Add $-\frac{6}{10}+\frac{5}{6}$

Algebra 1

It is also used when adding polynomials with unlike denominators.

Example 2: **Find** $\frac{a+1}{a}+\frac{a-3}{3a}$

Use the Bubble Method:



LCM: $3a$

$$\frac{a+1}{a}+\frac{a-3}{3a}$$
$$=\left(\frac{a+1}{a}\bullet\frac{3}{3}\right)+\frac{a-3}{3a}$$
$$=\frac{3a+3}{3a}+\frac{a-3}{3a}$$
$$=\frac{3a+3+a-3}{3a}$$
$$=\frac{4a}{3a}$$
$$=\frac{4\bullet \cancel{a}}{3\bullet \cancel{a}}$$
$$=\frac{4}{3}$$

You Try!

Add $\frac{4d^2}{d}+\frac{d+2}{d^2}$

In most books, when students learn to add or subtract unlike fractions, they are told to find the Lowest Common Denominator (LCD). But the LCD ***is*** really the LCM. As shown below, although the problems are different, the concept is similar, a whole number being added to either a mixed number or a rational expression. In both cases finding the LCM makes the problem much more accessible to the student.

Grades 5-7

Adding or Subtracting a Whole number and a Mixed Number.

Example 3: Subtract $5-2\frac{3}{5}$

Solving Using LCM:

$$5-2\frac{3}{5}$$
$$=\frac{5}{1}-\frac{13}{5}$$
$$=\left(\frac{5}{1}\bullet\frac{5}{5}\right)-\frac{13}{5}$$
$$=\frac{25}{5}-\frac{13}{5}$$
$$=\frac{25-13}{5}$$
$$=\frac{12}{5}$$
$$=2\frac{2}{5}$$

Solving Using Decomposition:

$$5-2\frac{3}{5}$$
$$=4\frac{5}{5}-2\frac{3}{5}$$
$$=\left(4+\frac{5}{5}\right)-\left(2+\frac{3}{5}\right)$$
$$=(4-2)+\left(\frac{5}{5}-\frac{3}{5}\right)$$
$$=2+\frac{2}{5}$$
$$=2\frac{2}{5}$$

You Try!

Solve $7-3\frac{2}{3}$

Algebra 1

Adding or Subtracting a Mixed Expression to a Rational Expression.

Example 3: Find $3+\frac{6}{\boldsymbol{x}+3}$

$$3+\frac{6}{x+3}$$
$$=\frac{3}{1}+\frac{6}{x+3}$$
$$=\left(\frac{3}{1}\bullet\frac{x+3}{x+3}\right)+\frac{6}{x+3}$$
$$=\frac{3x+9}{x+3}+\frac{6}{x+3}$$
$$=\frac{3x+9+6}{x+3}$$
$$=\frac{3x+15}{x+3}$$

$$1:1\bullet 1 \qquad x+3:x+3$$
$$1\bullet 1\bullet (x+3)$$

You Try!

Evaluate $r^2+\frac{r-4}{r+3}$

Grades 5-7

More fractions….

Example 4: Find

$$\left(\frac{2}{8}+\frac{3}{6}\right)-\left(\frac{11}{12}+\frac{5}{9}\right)$$

$$=\left[\left(\frac{2}{8}\bullet\frac{3}{3}\right)+\left(\frac{3}{6}\bullet\frac{4}{4}\right)\right]-\left[\left(\frac{11}{12}\bullet\frac{3}{3}\right)+\left(\frac{5}{9}\bullet\frac{4}{4}\right)\right]$$

$$=\left(\frac{6}{24}+\frac{12}{24}\right)-\left(\frac{33}{36}+\frac{20}{36}\right)$$

$$=\frac{18}{24}-\frac{53}{36}$$

$$=\left(\frac{18}{24}\bullet\frac{3}{3}\right)-\left(\frac{53}{36}\bullet\frac{2}{2}\right)$$

$$=\frac{54}{72}-\frac{106}{72}$$

$$=\frac{54-106}{72}$$

$$=-\frac{52}{72}$$

$$=\frac{-1\bullet\cancel{2}\bullet\cancel{2}\bullet 13}{\cancel{2}\bullet\cancel{2}\bullet 2\bullet 3\bullet 3}$$

$$=-\frac{13}{18}$$

You Try!

Solve $\left(\frac{1}{3}+\frac{1}{4}\right)-\left(\frac{3}{8}+\frac{1}{2}\right)$

Algebra 1

More Rational Expressions and Equations…

Example 4: Find

$$\frac{h-2}{h^2+4h+4}-\frac{h-4}{h^2-4}$$

$$=\frac{h-2}{(h+2)(h+2)}-\frac{h-4}{(h+2)(h-2)}$$

$$=\left(\frac{h-2}{(h+2)(h+2)}\bullet\frac{(h-2)}{h-2}\right)-\left(\frac{h-4}{(h+2)(h-2)}\bullet\frac{(h+2)}{(h+2)}\right)$$

$$=\frac{(h-2)(h-2)}{(h+2)(h+2)(h-2)}-\frac{(h-4)(h+2)}{(h+2)(h+2)(h-2)}$$

$$=\frac{h^2-4h+4-h^2-2h-8}{(h+2)(h+2)(h-2)}$$

$$=\frac{-2h+12}{(h+2)^2(h-2)}$$

$h^2-4h+4:(h+2)(h+2)$ $\quad$ $h^2-4:\cancel{(h+2)}(h-2)$

$$(h+2)(h+2)(h-2)$$

LCM: $(h+2)(h+2)(h-2)$

You Try!

$$\frac{k-3}{k^2+k-12}+\frac{k}{k^2-9}$$

Using the LCM through the grades allows for continuity for students. When students arrive at solving proportions or solving rational equations in Algebra 1, they can use the LCM as a tool to help solve.

Algebra 1

We can use the Lowest Common Multiple as one way to solve Proportions.

Example 5: Solve $\frac{4}{5}=\frac{8}{10x}$

What would be the LCM of 5 **and** $10x$?

$5: 1\bullet 5 \qquad 10x: 2\bullet \cancel{5}\bullet x$

$1\bullet 5\bullet 2\bullet x$

LCM:

$$\frac{4}{5}=\frac{8}{10x}$$

$$\frac{10x}{1}\bullet\frac{4}{5}=\frac{8}{10x}\bullet\frac{10x}{1}$$

$$\frac{2\bullet\cancel{5}\bullet x\bullet 4}{1\bullet\cancel{5}}=\frac{8\bullet\cancel{10}\bullet\cancel{x}}{\cancel{10}\bullet\cancel{x}\bullet 1}$$

$$\frac{8x}{1}=\frac{8}{1}$$

$$8x=8$$

$$x=1$$

You Try!

Solve $\frac{x}{x+4}=\frac{2}{x}$

Algebra I

Using Lowest Common Multiples to solve proportions helps prepare students to solve Rational Equations.

Example 6: Solve $\frac{1}{2}x-\frac{3}{4}=\frac{5}{12}x$

Solve Using Common Denominators:

$$\frac{1}{2}x-\frac{3}{4}=\frac{5}{12}x$$

$$\left(\frac{6}{6}\bullet\frac{1}{2}x\right)-\left(\frac{3}{3}\bullet\frac{3}{4}\right)=\frac{5}{12}x$$

$$\frac{6}{12}x-\frac{9}{12}=\frac{5}{12}x$$

$$\frac{6}{12}x-\frac{6}{12}x-\frac{9}{12}=\frac{5}{12}x-\frac{6}{12}x$$

$$-\frac{9}{12}=-\frac{1}{12}x$$

$$-\frac{12}{1}\bullet-\frac{9}{12}=-\frac{12}{1}\bullet-\frac{1}{12}x$$

$$9=x$$

Solve by Clearing the Denominator Using the LCM:

$$\frac{1}{2}x-\frac{3}{4}=\frac{5}{12}x$$

$$\left(12\bullet\frac{1}{2}x\right)-\left(12\bullet\frac{3}{4}\right)=\left(12\bullet\frac{5}{12}x\right)$$

$$\left(\frac{6\bullet\cancel{2}\bullet 1\bullet x}{\cancel{2}}\right)-\left(\frac{3\bullet\cancel{4}\bullet 3}{\cancel{4}}\right)=\left(\frac{\cancel{12}\bullet 5\bullet x}{\cancel{12}}\right)$$

$$6x-9=5x$$

$$6x-6x-9=5x-6x$$

$$-9=-x$$

$$\frac{-9}{-1}=\frac{-x}{-1}$$

$$9=x$$

You Try!

$\frac{3x}{5}+\frac{3}{2}=\frac{7x}{10}$

As a high school student, when I looked at problems such as the ones shown below, I would have cringed. Today I ask myself, "Where was the LCM when I needed it? Why didn't anyone teach me this?" The LCM is a valuable tool, not just for adding fractions in 6^{th} and 7^{th} grade, look how it can be used for Algebra II!

Algebra II

Example 1: Solve $\frac{1}{x}+\frac{1}{2x}=\frac{3}{x^2}$

Solve by clearing the denominators using the LCM.

$x:1\bullet x \quad 2x:2\bullet x \quad x^2:x\bullet x$

$1\bullet x\bullet 2\bullet x$

LCM: $2x^2$

$$\frac{1}{x}+\frac{1}{2x}=\frac{3}{x^2}$$

$$\frac{2x^2}{1}\bullet\frac{1}{x}+\frac{2x^2}{1}\bullet\frac{1}{2x}=\frac{2x^2}{1}\bullet\frac{3}{x^2}$$

$$\frac{2\bullet\cancel{x}\bullet x\bullet 1}{1\bullet\cancel{x}}+\frac{\cancel{2}\bullet\cancel{x}\bullet x\bullet 1}{\cancel{2}\bullet\cancel{x}}=\frac{2\bullet\cancel{x}\bullet\cancel{x}\bullet 3}{1\bullet\cancel{x}\bullet\cancel{x}}$$

$$2x+x=6$$

$$3x=6$$

$$x=2$$

Algebra II

Example 2: Solve $\frac{1}{x-2}+\frac{1}{x+2}=\frac{2}{x^2-4}$

Solve by clearing the denominators using the LCM

$x-2:(x-2) \quad x+2:(x+2) \quad x^2-4:\cancel{(x-2)}\cancel{(x+2)}$

$(x-2)(x+2)$

LCM: $(x-2)(x+2)$

$$\frac{1}{x-2}+\frac{1}{x+2}=\frac{2}{x^2-4}$$

$$\frac{(x-2)(x+2)}{1}\bullet\frac{1}{x-2}+\frac{(x-2)(x+2)}{1}\bullet\frac{1}{x+2}=\frac{(x-2)(x+2)}{1}\bullet\frac{2}{x^2-4}$$

$$\frac{(x-2)(x+2)\bullet 1}{1\bullet x-2}+\frac{(x-2)(x+2)\bullet 1}{1\bullet x+2}=\frac{(x-2)(x+2)\bullet 2}{1\bullet(x-2)(x+2)}$$

$$x+2+x-2=2$$

$$2x=2$$

$$x=1$$

You Try!

$\frac{1}{w}+\frac{6}{w^2}=1$

Algebra II	**Algebra II**
The Lowest Common Multiple can also be used as an efficient way to solve Complex Fractions. **Example 3: Solve** $\dfrac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{6}}$ $\dfrac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{6}}$ $=\dfrac{\left(\frac{1}{2}+\frac{1}{3}\right)\bullet\frac{6}{1}}{\left(1-\frac{1}{6}\right)\bullet\frac{6}{1}}$ $=\dfrac{3+2}{6-1}$ $=\dfrac{5}{5}$ $=1$ **You Try!** Evaluate $\dfrac{\frac{2}{3}-\frac{5}{6}}{\frac{1}{3}+\frac{2}{9}}$	**Example 4: Solve** $\dfrac{\frac{3\boldsymbol{x}}{4}}{\frac{\boldsymbol{x}}{2}+\frac{\boldsymbol{x}}{5}}$ $\dfrac{\frac{3x}{4}}{\frac{x}{2}+\frac{x}{5}}$ $=\dfrac{\left(\frac{3x}{4}\right)\bullet\frac{20}{1}}{\left(\frac{x}{2}+\frac{x}{5}\right)\bullet\frac{20}{1}}$ $=\dfrac{15x}{10x+4x}$ $=\dfrac{15x}{14x}$ $=\dfrac{15}{14}$ **You Try!** **Evaluate** $\dfrac{z-\frac{1}{z}}{1-\frac{1}{z}}$

Worked Out You Try Solutions:

Grades 5-7

You Try 1: Find the LCM of $12, 16, \textbf{and} 10$

$12: 2 \bullet 2 \bullet 3 \quad 16: \not{2} \bullet \not{2} \bullet 2 \bullet 2 \quad 10: \not{2} \bullet 5$

$2 \bullet 2 \bullet 3 \bullet 2 \bullet 2 \bullet 5$

∴ The LCM of 12, 16 and 10 is 240

You Try 2: Add $-\frac{6}{10} + \frac{5}{6}$

$-\frac{6}{10} + \frac{5}{6}$

$= \left(-\frac{6}{10} \bullet \frac{3}{3}\right) + \left(\frac{5}{6} \bullet \frac{5}{5}\right)$

$= -\frac{18}{30} + \frac{25}{30}$

$6: 2 \bullet 3 \quad 10: 2 \bullet 5$

$2 \bullet 3 \bullet 5$

$= \frac{-18 + 25}{30}$

$= \frac{7}{30}$

You Try 3: **Solve** $7 - 3\frac{2}{3}$

$7 - 3\frac{2}{3}$

$= \frac{7}{1} - \frac{11}{3}$

$= \left(\frac{7}{1} \bullet \frac{3}{3}\right) - \frac{11}{3}$

$= \frac{21}{3} - \frac{11}{3}$

$= \frac{21 - 11}{3}$

$= \frac{10}{3} = 3\frac{1}{3}$

Algebra 1

You Try 1:
Find the LCM of $28m^2n$ **and** $12m^2n^3p$

$28m^2n$: $2 \bullet 2 \bullet 7 \bullet m \bullet m \bullet n$

$12m^2n^3p$: $\not{2} \bullet \not{2} \bullet 3 \bullet \not{m} \bullet \not{m} \bullet \not{n} \bullet n \bullet n \bullet p$

$2 \bullet 2 \bullet 7 \bullet m \bullet m \bullet n \bullet 3 \bullet n \bullet n \bullet p$

∴ The LCM is $81m^2n^3p$

You Try 2: Add $\frac{4d^2}{d} + \frac{d+2}{d^2}$

$\frac{4d^2}{d} + \frac{d+2}{d^2}$

$= \left(\frac{4d^2}{d} \bullet \frac{d}{d}\right) + \frac{d+2}{d^2}$

$= \frac{4d^3}{d^2} + \frac{d+2}{d^2}$

$= \frac{4d^3 + d + 2}{d^2}$

You Try 3: **Evaluate** $r^2 + \frac{r-4}{r+3}$

$r^2 + \frac{r-4}{r+3}$

$= \left(\frac{r^2}{1} \bullet \frac{r+3}{r+3}\right) + \frac{r-4}{r+3}$

$= \frac{r^3 + 3r^2 + r - 4}{r+3}$

Grades 5-7

You Try 4: Solve $\left(\frac{1}{3}+\frac{1}{4}\right)-\left(\frac{3}{8}+\frac{1}{2}\right)$

$$\left(\frac{1}{3}+\frac{1}{4}\right)-\left(\frac{3}{8}+\frac{1}{2}\right)$$

$$=\left[\left(\frac{1}{3}\bullet\frac{4}{4}\right)+\left(\frac{1}{4}\bullet\frac{3}{3}\right)\right]-\left[\frac{3}{8}+\left(\frac{1}{2}\bullet\frac{4}{4}\right)\right]$$

$$=\left(\frac{4}{12}+\frac{3}{12}\right)-\left(\frac{3}{8}+\frac{4}{8}\right)$$

$$=\frac{7}{12}-\frac{7}{8}$$

$$=\left(\frac{7}{12}\bullet\frac{2}{2}\right)-\left(\frac{7}{8}\bullet\frac{3}{3}\right)$$

$$=\frac{14}{24}-\frac{21}{24}$$

$$=\frac{14-21}{24}$$

$$=-\frac{7}{24}$$

Algebra 1

You Try 4: $\frac{k-3}{k^2+k-12}+\frac{k}{k^2-9}$

$$\frac{k-3}{k^2+k-12}+\frac{k}{k^2-9}$$

$$=\frac{k-3}{(k-3)(k+4)}+\frac{k}{(k+3)(k-3)}$$

$$=\frac{k-3}{(k-3)(k+4)}\bullet\frac{(k+3)}{(k+3)}+\frac{k}{(k+3)(k-3)}\bullet\frac{(k+4)}{(k+4)}$$

$$=\frac{k^2-9}{(k-3)(k+4)(k+3)}+\frac{k^2+4k}{(k-3)(k+4)(k+3)}$$

$$=\frac{k^2-9+k^2+4k}{(k+4)(k+3)(k-3)}$$

$$=\frac{2k^2+4k-9}{(k+4)(k+3)(k-3)} \text{ or } \frac{2k^2+4k-9}{k^3-9k+4k^2-36}$$

You Try 5: **Solve** $\frac{x}{x+4}=\frac{2}{x}$

$$\frac{x}{x+4}=\frac{2}{x}$$

$$x(x+4)\bullet\frac{x}{x+4}=\frac{2}{x}\bullet x(x+4)$$

$$\frac{x\bullet\cancel{(x+4)}\bullet x}{\cancel{x+4}}=\frac{2\bullet\cancel{x}\bullet(x+4)}{\cancel{x}}$$

$$x\bullet x=2\bullet(x+4)$$

$$x^2=2x+8$$

$$x^2-2x-8=0$$

$$(x+2)(x-4)=0$$

$$x+2=0 \quad x-4=0$$

$$x=-2 \quad x=4$$

Algebra 1

You Try 6: $\frac{3x}{5}+\frac{3}{2}=\frac{7x}{10}$

$$\frac{3x}{5}+\frac{3}{2}=\frac{7x}{10}$$

$$10\bullet\frac{3x}{5}+10\bullet\frac{3}{2}=\frac{7x}{2}\bullet 10$$

$$6x+15=7x$$

$$6x-6x+15=7x-6x$$

$$15=x$$

Algebra II

You Try 1: $\frac{1}{w}+\frac{6}{w^2}=1$

$$\frac{1}{w}+\frac{6}{w^2}=1$$

$$w^2\bullet\frac{1}{w}+w^2\bullet\frac{6}{w^2}=1\bullet w^2$$

$$w+6=w^2$$

$$w^2-w-6=0$$

$$(w+2)(w-3)=0$$

$$w+2=0 \quad w-3=0$$

$$w=-2 \quad w=3$$

You Try 2: Evaluate $\frac{\frac{2}{3}-\frac{5}{6}}{\frac{1}{3}+\frac{2}{9}}$

$$\frac{\frac{2}{3}-\frac{5}{6}}{\frac{1}{3}+\frac{2}{9}}$$

$$=\frac{18\bullet\frac{2}{3}-18\bullet\frac{5}{6}}{18\bullet\frac{1}{3}+18\bullet\frac{2}{9}}$$

$$=\frac{12-15}{6+4}$$

$$=-\frac{3}{10}$$

Algebra II

You Try 3: **Evaluate** $\dfrac{z-\frac{1}{z}}{1-\frac{1}{z}}$

$$\frac{z-\frac{1}{z}}{1-\frac{1}{z}}$$

$$=\frac{\left(z-\frac{1}{z}\right)\bullet z}{\left(1-\frac{1}{z}\right)\bullet z}$$

$$=\frac{z^2-1}{z-1}$$

$$=\frac{(z+1)\cancel{(z-1)}}{\cancel{z-1}}$$

$$=z+1$$